BLUFFTON UNIVERSITY

MATHEMATICS CONTEST 2019
PART I: 30 Minutes
NO CALCULATORS

Section A. Each correct answer is worth 1 point.

1. Simplify: $2^2 + 3^1 + 4x^0 + (5x)^0$

2. Give the geometric name of the line segment that connects two points on a circle.

3. On your answer sheet, circle the letters of all sets of numbers which are ***closed under addition***:
   A) $\{-1, 0, 1\}$ B) Odd integers C) Even integers
   D) Prime numbers E) $\{\ldots, -3, -2, -1, 1, 2, 3, \ldots\}$

4. Let a, b, c, d be the numbers $2, 0, 1, 9$ (all four numbers, but not necessarily in that order). What is the ***smallest positive rational number*** that can be expressed by $\dfrac{a-b}{c-d}$?

5. If $f(x) = x^2 + 2$, and $g(x) = x - 1$, find the value of $f(g(3)) - g(f(3))$.

6. The average of 3 and 7 (two distinct prime numbers) is also a prime number—namely, 5. Find two more distinct prime numbers, both less than 17, such that their average is also a prime number.

7. Solve for all real values of x: $|x - 2019| = 1000$

Section B. Each correct answer is worth 2 points.

8. What number, decreased by 2019, is 4 times itself?

9. Find the solution—expressed as an ordered pair (x, y)—to the system $\begin{cases} 3x + 5y = 20 \\ 5x - 6y = 19 \end{cases}$

10. Write an equation for the line passing through the vertices of the ellipse $\dfrac{(x-2018)^2}{9} + \dfrac{(y+2019)^2}{4} = 1$.

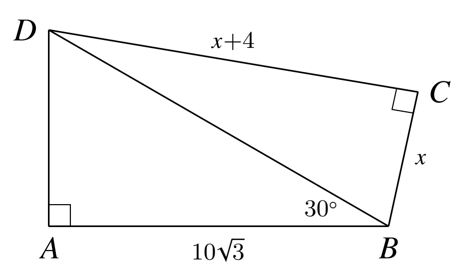
11. Choose (at random) a number from the set of ***negative even two-digit*** integers, and (also at random) a number from the set of ***positive odd three-digit*** integers. What is the probability (expressed as a percent) that the product of their squares is ***odd***?

12. Convert $(5\sqrt{2}, 135°)$ from polar coordinate form (r, θ) to rectangular coordinate form (x, y).

Section C. Each correct answer is worth 3 points.

13. Given: $a + b + c = 14$, $ab = 14$, and $c^2 = a^2 + b^2$. Find the numerical value of c.

14. For quadrilateral $ABCD$ shown on the right, find the exact value of the perimeter.



15. Simplify, writing your answer in the form $\boldsymbol{a + bi}$:
$$\frac{2020 + 2020i + 2019i^2 + 2019i^3}{1 - i}$$

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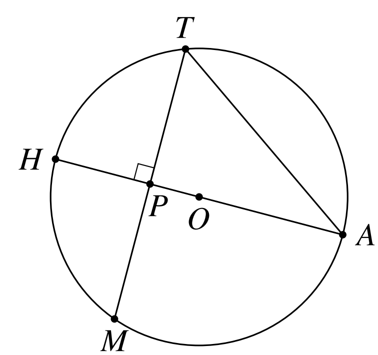
MATHEMATICS CONTEST 2019
PART II: 30 Minutes
CALCULATORS ALLOWED

***Section A.* Each correct answer is worth 1 point.**

1. The area of a rectangle, with integer sides, is 2019 square centimeters. Find ***all possible integral values*** for its perimeter.
2. The average of 19 numbers is 20. When one number is dropped, the average of the remaining set of numbers is 19. What number was dropped?
3. For ***how many*** integral values of x does there exist a triangle whose sides have length $6, 8$, and x?
4. Express as a ratio in simplest form: $\dfrac{\text{19 hours, 30 minutes}}{\text{20 hours, 30 minutes}}$
5. Subtract and express your answer using Roman numerals: MMXIX – CCCLIII.
6. Write the exact value of $\left|\sqrt{2} - \pi\right|$.
7. Over a number of years, the Beavers played 250 games and won 8 more games than they lost. Find the percent of the games they won. Express your answer to the nearest tenth of one percent.

***Section B.* Each correct answer is worth 2 points.**

8. What is the ***exact*** area of a triangle whose sides are $19, 20$, and 19?
9. Assuming all angles acute, solve for θ (in degrees): $\sin(20°) = \cos(2\theta)$.
10. You are looking for integers n such that both $n + 100$ and $n + 164$ are perfect squares. One such integer is $n = 125$, because
    $125 + 100 = 225 = 15^2$ and $125 + 164 = 289 = 17^2$.
    There are two other integers n less than 125 with this property. Find ***one*** of those integers.
11. Given circle O as shown, with $HP = 4$ and $TM = 16$. Find the value of TA, correct to the nearest hundredth.
12. The product of two ***consecutive negative even integers*** is 5 more than 2019. Find the ***sum*** of these two integers.



***Section C.* Each correct answer is worth 3 points.**

13. Flying with a steady tailwind, a plane flies 450 miles from airport M to airport N in $2\frac{1}{2}$ hours. With the wind remaining steady as before, the return trip to airport M takes 3 hours, 20 minutes. What is the wind speed?
14. Factor completely over the rational numbers: $a^4 - 121 - 16a^3 + 64a^2$
15. Basketball player Dakota currently has made $\boldsymbol{A}$ free throws in $\boldsymbol{B}$ attempts, and his/her free throw shooting percentage is less than 50%. How many consecutive free throws must Dakota make in order to raise her/his average to exactly 50%? Express your answer in terms of $\boldsymbol{A}$ and $\boldsymbol{B}$.