BLUFFTON
EMBRACING SPIRIT, ENGAGING MINDS

MATHEMATICS CONTEST 2014
PART I: 30 Minutes
NO CALCULATORS

Section A. Each correct answer is worth 1 point.

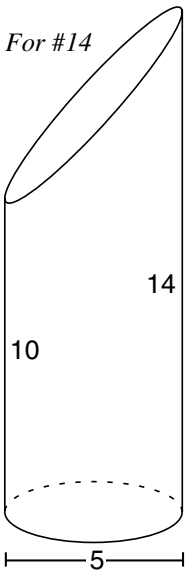
1. Evaluate: $(20 - 14)(14 - 20)$.
2. $\sqrt[3]{300}$ is between what two consecutive integers?
3. Player P scored 3 points, Player Q scored three times as many as Player P, Player R scored 3 more points than Player P, Player S scored twice as many points as Player Q, and Player T scored 13 points. How many total points were scored by Players P, Q, R, S, and T?
4. Last year we had 20. This year we have 15. What is the percent of decrease?
5. The measure of an angle is 54°. Find the average of its supplement and twice its complement.
6. How many positive integer solutions exist for $3(x - 8) \leq 5$?
7. Three integers are chosen at random from $\{1, 2, 3, \ldots, 1000\}$. (Repetition is allowed; that is, a number may be chosen more than once.) What is the probability that their sum is *even*? Express your answer as a *percent*.

Section B. Each correct answer is worth 2 points.

8. Simplify the fraction: $\dfrac{m^3 + 2m^2 - 8m}{m^2 - 4}$
9. Find the vertex of the parabola $y = x^2 - 20x + 14$. Write as an ordered pair (h, k).
10. A rectangle with whole-number dimensions has an area of 72 square units. Let M be the maximum perimeter for such a rectangle, and m be the minimum perimeter. What is $M - m$?
11. Find y so that the points $A(0, 0)$, $B(2, 2)$ and $C(-4, y)$ are the vertices of a right triangle with hypotenuse $\overline{AC}$.
12. The polynomial equation $x^3 + bx^2 + cx - 18 = 0$ has real coefficients, and the complex number $3i$ is a root. Find the values of b and c.

Section C. Each correct answer is worth 3 points.

13. Express as a ratio of two integers in simplest form: $\dfrac{\left(\frac{2}{3}\right)^{-2} + \left(\frac{3}{2}\right)^{-3}}{3^{-2}}$
14. (See figure.) Find the exact volume of the cylinder cut on a slant.



15. Solve for x if $x - 5 = \det \begin{vmatrix} \log_4 32 + \log_3 27 + \log_9 3 & \dfrac{(11-3)!}{5! \cdot 6 \cdot 7} \\ 2\cos(0^\circ) - \sin(0^\circ) & \displaystyle\sum_{k=-1}^{2} |k| \end{vmatrix}$

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MATHEMATICS CONTEST 2014
PART II: 30 Minutes
CALCULATORS NEEDED

Section A. Each correct answer is worth 1 point.

1. $(20+14)(15+16) = (14+20)(15+16)$ is an example of the _____ property of addition. Circle the letter of the term that correctly fills in this blank.
   A) associative B) commutative C) distributive D) identity E) inverse
2. Driving at a constant rate of speed, Danica drove 180 miles in 3 hours. She then increased her rate 5 mph and drove 2 more hours. What was the total distance that she drove?
3. What is the name of an angle whose vertex is the center of a circle, and whose sides are radii of that circle?
4. The set of seven natural numbers { 6, ___, 4, 9, ___, 8, 3 } has mean 7 and mode 4. What are the two missing numbers? (Order does not matter.)
5. A trapezoid has height 10 inches, midsegment (median) of length 15 inches, and bases of x inches and $(x+8)$ inches. Find x.
6. Given $\triangle RST \sim \triangle LNE$ with $RT = 10, TS = 9, RS = 8, EN = 6x-12$, and $LE = 5x+5$, find the numerical value of LN.
7. Find the slope of any line that is perpendicular to the line $x - 20 = 14$.

Section B. Each problem is worth 2 points.

8. List all the integers from 25 to 30, inclusive, that cannot be expressed as the sum of two squares. (For example, $5 = 2^2 + 1^2$ can be expressed as the sum of two squares, but 6 cannot be expressed this way.)
9. On the answer sheet, circle the letter of the one expression which is not a factor of $x^6 - y^6$.
   A) $x^2 - y^2$ B) $x^3 + y^3$ C) $x^4 + x^2y^2 + y^4$
   D) $x^2 + 2xy + y^2$ E) $x^3 - 2x^2y + 2xy^2 - y^3$
10. If $2^3(20m + 14n) - 3^2(14m - 20n) = 2014n$, find the ratio of m to n, expressed as a ratio of two integers in simplest form.
11. Convert from base 8 to base 5: $403_8 =$ _____ $_5$.
12. Subtract the opposite of $(14x^2 - 20x + 7^2)$ from $(20x^2 + 4^3 + 14x + 5^0)$, and simplify.

Section C. Each problem is worth 3 points.

13. List all prime numbers between 100 and 199 such that the tens digit is a prime number, the units digit is a prime number, and the tens and units digit taken together are a two-digit prime number.
14. Given: $f(x) = x^2 - 20x + 11$ and $g(x) = \sqrt{x+4}$. If $g(x) = 5$, find $g(f(x))$.
15. Let $5, m, 20, \ldots$ be an arithmetic sequence. Let $5, n, 20, \ldots$ (with $n \geq 0$) be a geometric sequence. Let $5, p, q, 20, \ldots$ be a Fibonacci-style sequence. Write the value of $m - n + p$.