BLUFFTON
EMBRACING SPIRIT, ENGAGING MINDS

MATHEMATICS CONTEST 2005
PART I: 30 Minutes
NO CALCULATORS

Section A. Each correct answer is worth 1 point.

1. List all positive integral factors of 20.
2. Evaluate: $2 \cdot 0 - 0 + 5$
3. By definition, a ___?___ is a quadrilateral with two pairs of parallel sides.
4. Which one of the following is not a valid postulate or theorem to establish triangle congruence?
   A) SSS B) AAA C) SAS D) ASA E) AAS
5. The area of a square is 100 cm^2. What is its perimeter (including correct units)?
6. If there are 12 4-cent stamps in a dozen, how many 3-cent stamps are in a dozen?
7. The year 2002 was a palindrome (a word or number that reads the same forward as backward). How many palindromic years were there between 1500 and 2001?

Section B. Each correct answer is worth 2 points.

8. Using any of the ten digits, 0–9, but with no repeats, write the smallest possible 6-digit positive even integer with a "2" in the thousands place.
9. Express in simplest $a + bi$ form: $i^{2005} + 2005i + i^{-2005} + 2005$.
10. Which of the following expressions is not equal to the other four?
    A) $\dfrac{a+b}{-(a+b)}$ B) $\dfrac{a-b}{b-a}$ C) $\dfrac{b+a}{-a-b}$ D) $\dfrac{a+b}{a-b}$ E) $-\dfrac{a+(-b)}{-b+a}$
11. Solve for m: $\begin{bmatrix} 6 & n \\ m & p \end{bmatrix} = 3 \begin{bmatrix} 2 & x \\ 12 & y \end{bmatrix}$
12. Let $f(x) = x^2 - x - 1$. If $f(a-1) = 1$, solve for a.

Section C. Each correct answer is worth 3 points.

13. The hyperbola $\dfrac{(x+1)^2}{9} - \dfrac{(y-2)^2}{25} = 1$ has two asymptotes. Give the equation of the asymptote with a positive slope, in the form $y = mx + b$.
14. You are rolling two (six-sided fair) dice. Let A be the probability of rolling a sum of 8, and let B be the probability of rolling a sum of 5. Give the exact value of $A - B$.
15. Factor completely: $a^4 + 16a^3 + 64a^2 - 81$

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MATHEMATICS CONTEST 2005
PART II: 30 Minutes
CALCULATORS NEEDED

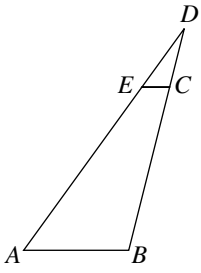
Section A. Each correct answer is worth 1 point.

1. Write the prime factorization of 2005.
2. The perimeter of a square is 100 feet. What is its area (including correct units)?
3. Which one of the following is not a solution to $3x - 2y = -24$?
   A) $(-8, 0)$ B) $(8, -24)$ C) $(-5, 4.5)$ D) $(0, 12)$ E) $(-2, 9)$
4. What is the answer when the sum of 2005, 4010, and 8020 is subtracted from the product of 200.5 and 401.0?
5. Write the numerical value of $\log_2 8 - \log_4 8 + \log_8 8$.
6. Suppose that you have an inexhaustible supply of 4-cent and 7-cent stamps. List all the amounts between 1 cent and 1 dollar, inclusive, that cannot be made using these stamps.
7. Solve for all real numbers: $x^3 = 64x$

Section B. Each problem is worth 2 points.

8. If $(3x - 2)^6$ is expanded, write the coefficient of x^3.
9. Given: the geometric sequence $2, a, b, 250$. Find the sum of a and b.
10. Evaluate: $(\sum_{k=0}^{3} k)! - \sum_{k=0}^{3} (k!)$
11. The clock on the wall of the mathematics classroom is a standard clock with an hour hand and a minute hand. What is the measure in degrees of the small angle formed at 10:45?
12. Solve for b_1: $A = \frac{1}{2}h(b_1 + b_2)$

Section C. Each problem is worth 3 points.



13. Given: $\triangle ABD$ with chord $\overline{EC}$ parallel to side $\overline{AB}$. The area of $\triangle ABD$ is 900 square units. If the ratio $DC : CB$ is $1 : 3$, find the area of trapezoid $ABCE$.
14. Three mathematics teachers are grading their students' tests.
    Ms. Rosie Kieffer's 27 students averaged exactly 82.
    Ms. Pam Riffle's 28 students averaged exactly 81.
    Mr. Chris Leuthold's 29 students averaged exactly 80.
    To the nearest hundredth, find the average score for all the students.
15. In a triangle, the largest angle is twice the size of the smallest angle. The lengths of the sides are consecutive integers. Find the sine of the smallest angle.