Some Beta-Decay Basics

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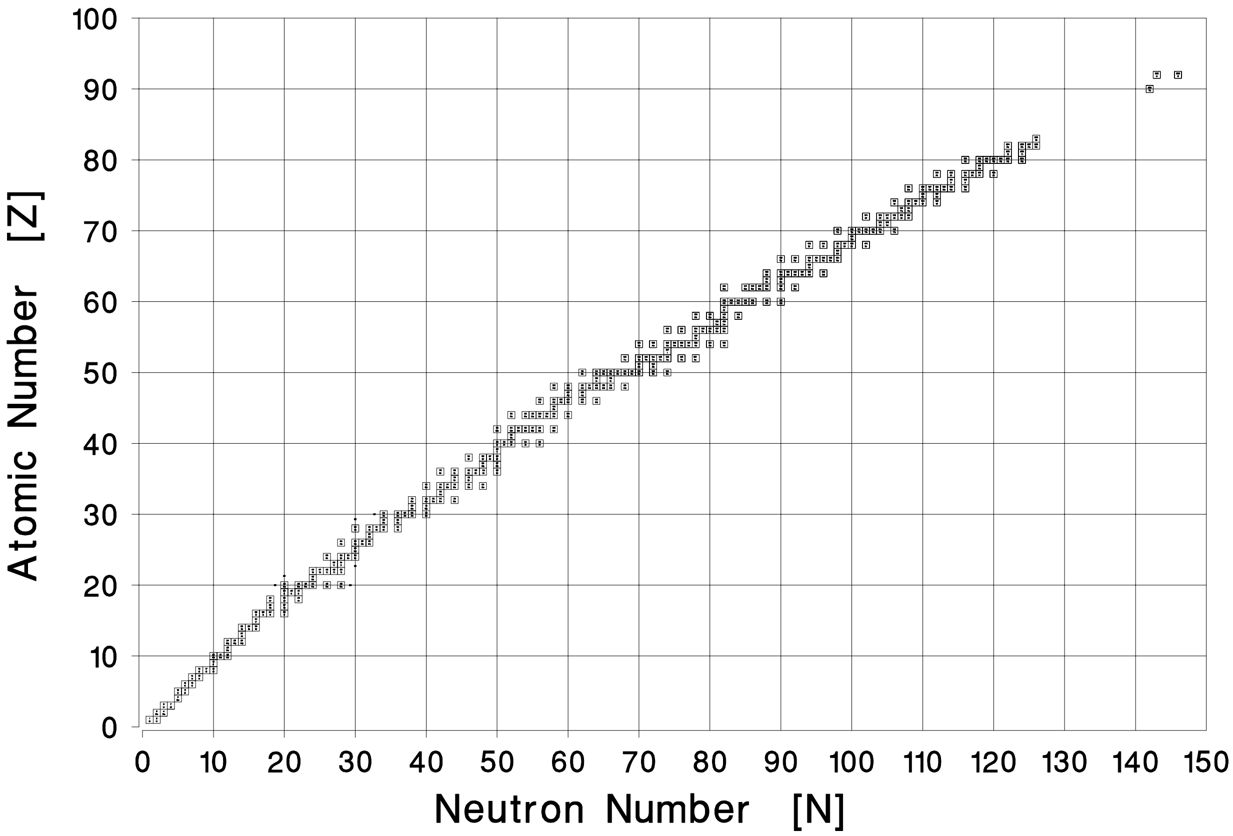
The figure at bottom of the page is known as a *Segrè Chart*. The Segrè Chart is one way to arrange and observe information about nuclei. A Segrè chart is simply a display of nuclides arranged with the number of neutrons increasing horizontally and the number of protons increasing vertically. It is common to see this type of chart enlarged and displayed on the wall in nuclear facilities and labs. For many years General Electric Company compiled and printed of Segrè charts under the name *Chart of the Nuclides*. Today Lockheed Martin has taken over compilation and printing of the chart. The website is www.ChartOfTheNuclides.com.

A tremendous amount of nuclear physics can be learned from the chart. Some info is observed using the broad view as shown below while other info requires a close-up view. Let's begin with the overall view.

The chart below is an excerpt of the full chart. It contains only *stable* nuclides whereas the complete chart shows all known nuclides. The reason for limiting our chart to stable nuclides is so we can get an idea of why certain nuclides are stable. The most obvious lesson is the proton-neutron ratio is strictly limited. For example, it is not possible to have a stable nucleus with 40 protons and 10 neutrons. Apparently that would be too many protons. Indeed, every combination above and to the left of the *band of stability* has too many protons. Likewise, every combination below and to the right of the band of stability has too many neutrons.

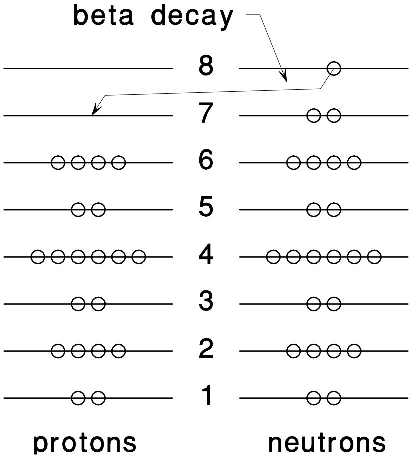
The band of stability begins with roughly equal numbers of protons and neutrons. However, by the time we get to 20 protons and 20 neutrons the trend toward more neutrons than protons is beginning to be obvious. At 50 protons the most favorable number of neutrons is about 70. At 80 protons the most favorable number of neutrons is abut 120.



How do we interpret this? Protons repel by Coulomb repulsion so some other force besides the electrostatic force must be active in the nucleus. The primary nuclear force is known as the *nuclear strong force*. This force is attractive and exists between protons and protons, between neutrons and neutrons, and between protons and neutrons. In other words, the strong force is an *inter-nucleon force* where nucleon refers to either a proton or a neutron. If we view the electrostatic repulsion and the nuclear strong force in a tug-of-war, stable nuclei must be those in which the nuclear strong force is winning. Adding neutrons ought to increase the inter-nucleon strong force attractions without increasing the electrostatic repulsions. This allows us to hypothesize that it makes sense for stable nuclides to contain more neutrons than protons. Perhaps too many protons allows the electrostatic repulsion to win. This cannot be the whole picture or any large number of neutrons ought to be stable. However, there are no stable nuclides consisting of only neutrons, and there is clearly a limit on the number of neutrons that can be stable with any specific number of protons.

Three pieces of new information help us understand what's going on. We introduce these at this point as known facts and we learn the evidence for them later. (1) Neutrons can be turned into protons and protons can be turned into neutrons. Conversion between protons and neutrons is what is taking place in the nuclear decay process known as *beta decay*. (2) Neutrons and protons exist in quantized energy levels very similar to electronic orbitals in atoms. Each energy level can contain a certain number of nucleons. The energy levels for protons and for neutrons are distinct; i.e. there are two sets of energy levels. (3) There are other forces at play in addition to the coulomb force and the nuclear strong force. One that plays an important role in beta decay is the *nuclear weak force*.

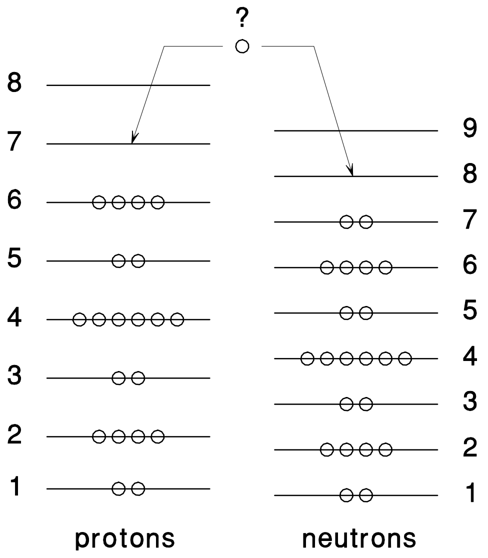
In the figure on the right we see some hypothetical quantum states for protons and neutrons in a nucleus. As shown, the nucleus has more neutrons than protons. Let's assume level-1can hold two nucleons, level-2 can hold four nucleons, etc. as shown. If level-7 can hold two nucleons, then when the 23rd neutron is added it has to go into level-8. If it had been a proton it could have gone into level-7 on the proton side and that would be lower energy. This being the case, the neutron changes into a proton so the overall energy is lower than it would be if the neutron had not changed.



This should make it clear that if (1) neutron/proton interconversion can occur, and (2) there is a quantum structure with fixed limits on the number of nucleons in each level, then beta decay will force the stable nuclides to have nearly equal numbers of protons and neutrons.

The quantum structure makes it energetically unfavorable to have too many of one type of nucleon compared to having a more balanced number of each type of nucleon. Even so, we could imagine nuclides with large imbalances if it were not possible for protons and neutrons to interconvert via beta decay. But since beta decay can occur, large imbalances cannot be stable; they will beta decay to a more stable configuration. That being the case, why is there a gradual shift towards more neutrons for the more massive stable nuclides? This is where the electrostatic repulsion comes into play.

Electrostatic repulsion between the protons makes those quantum levels slightly more spread out and raised in energy compared to the neutron levels. This is shown in the next figure. As can be seen in this hypothetical case, level-5 for protons is higher than level-6 for neutrons; level-6 for protons in higher than level-7 for neutrons, etc. As we keep going up in quantum levels, the proton levels keep getting high and higher in energy compared to the same-numbered neutron levels because of the electrostatic repulsion between the protons.

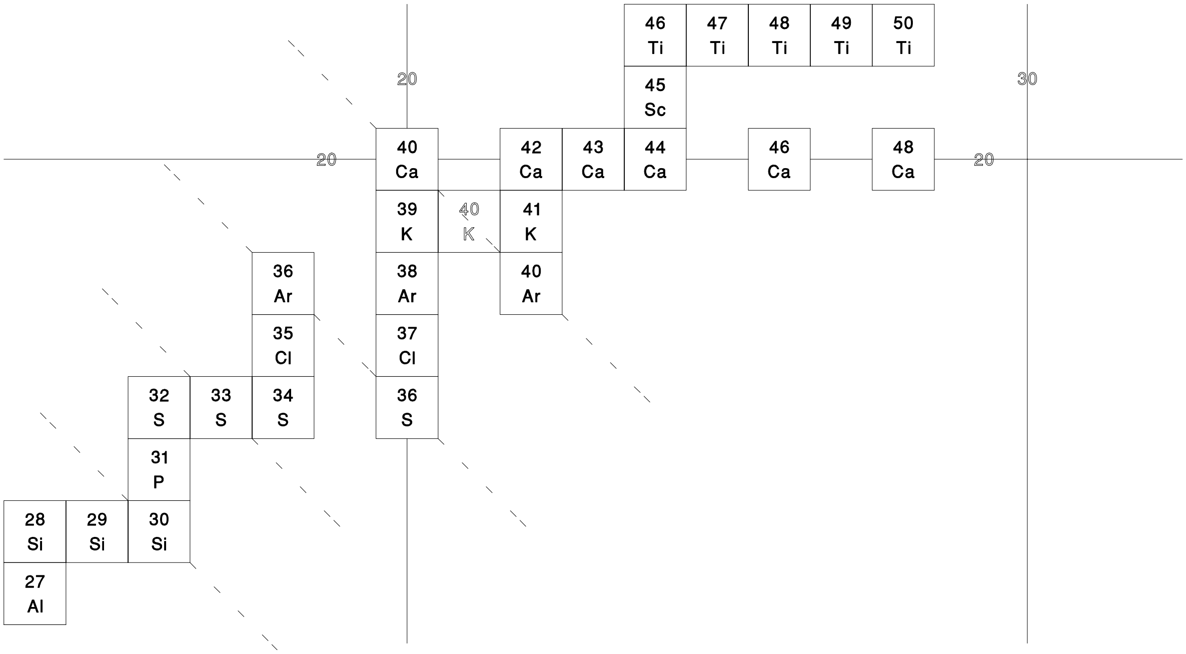


This nucleus already has 20 protons and 22 neutrons. The figure shows a new nucleon being added. One might assume the next nucleon ought to be a proton. However, a new proton would have to go into quantum level-7 for protons and a new neutron would have to go into quantum level-8 for neutrons. (The new nucleon cannot go into lower levels for either protons or neutrons because those levels are assumed to be already filled.) Given this situation the new nucleon should be a neutron even though the nucleus already has more neutrons than protons. In fact, the next several nucleons added have to be neutrons until neutron quantum level-8 is filled. Only then can we begin to put protons in quantum level-7 and expect the nucleus to be stable toward beta decay.

Therefore we indeed reason that Coulomb repulsion is responsible for the fact that stable nuclides have more neutrons than protons (once we get past ^{40}Ca). But it takes more knowledge of quantum structure and knowledge of beta decay (and weak nuclear force) to fully appreciate the balance between protons and neutrons.

A Closer Look at the Segré Chart

The figure at the bottom of this page is an enlargement of a small part of the full chart. The segment shown is in the vicinity of 20 protons and 20-30 neutrons. An obvious thing to notice is that some elements (aluminum, phosphorous and scandium) appear only once each while others appear multiple times. That means there is only one stable *isotope* for Al, P, and Sc. But silicon has three stable isotopes as does argon. It might appear that potassium has three stable isotopes, but this is not true. ^{40}K is radioactive and does not truly belong on a chart that supposedly contains only stable nuclides. But ^{40}K is so close to being stable that its half-life is 1.3 billion years. That means it is naturally occurring because a little ^{40}K from creation is still



around. There are several elements like this, and it is common to include them in charts that are supposed to include only stable nuclides. When this is the case they are usually marked in some way; in this chart ^{40}K is shown with hollow letters.

Notice that calcium (Z = 20) has an unusually high number of stable isotopes. When looking for elements with high number of stable isotopes we are looking horizontally in the Segré Chart for *rows* with lots of squares. If we look at the whole chart we see another large row (Z = 50) which is tin. Tin, with ten stable isotopes, has the most stable isotopes of all elements.

Now look at the *columns* on the chart (vertically). Start with the enlarged section. Nuclides with the same number of neutrons are called *isotones*. Many times there are one, two or three isotones for a given neutron number, but for N = 20 there are five isotones. This is interesting, 20 protons yields the highest number of stable nuclides in this region of the chart, and 20 neutrons also yields the highest number of stable nuclides.

Now look at the whole chart and look for columns that are longer than their neighbors. Besides 20, other standouts are 28, 50, and 82. These numbers, as well as the numbers 2, 8, and 126 have become known as *magic numbers*. The idea is that we are seeing some sort of extra stability when the proton number or the neutron number is one of these magic numbers. This is taken for evidence of a shell phenomenon in nuclear quantum levels similar to what we observe as periodicity in the periodic table. The magic numbers of 2, 8, 20, 28, 50, 82, 126 represent the total number of a particular nucleon present when a shell (quantum level) for that nucleon becomes full.

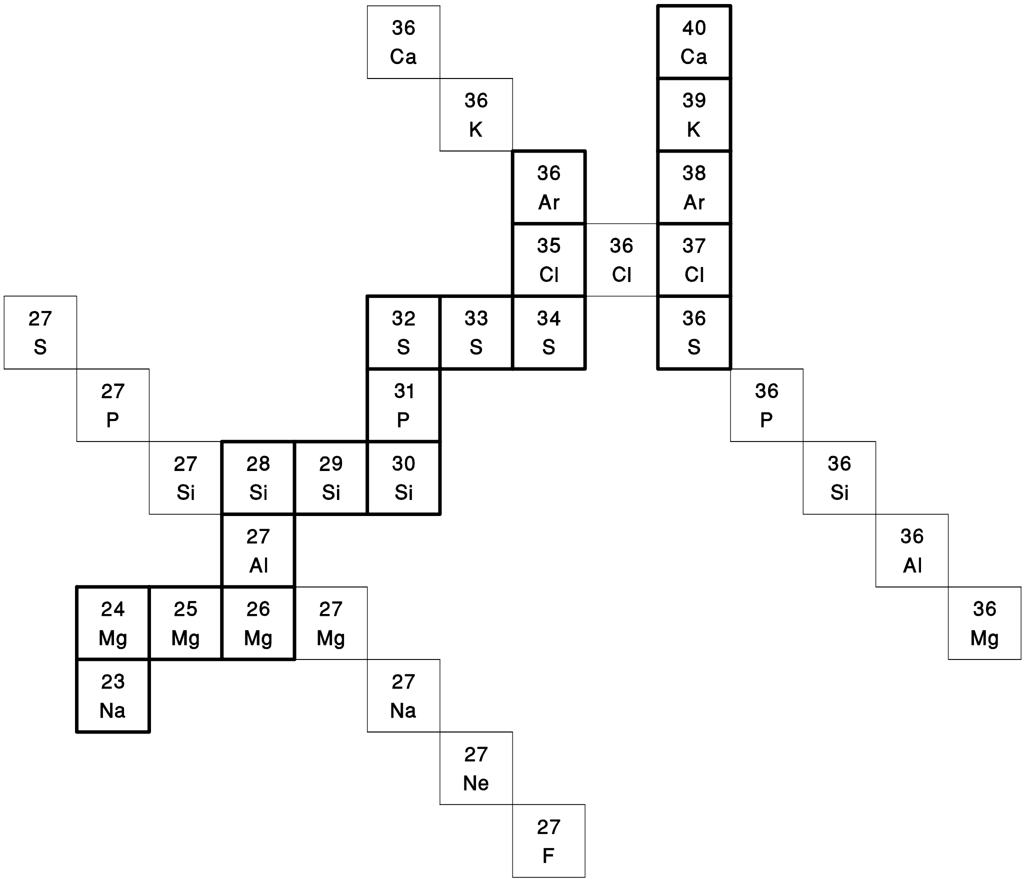
A nucleus with a magic number of protons or neutrons is called a magic nucleus. There are four *doubly magic* nuclides: ^{4}He (2,2),^{16}O (8,8), ^{40}Ca (20,20) and ^{208}Pb (82, 126). These are especially stable nuclides.

Now look at the enlarged chart and notice the dotted lines. These diagonal lines represent *isobars*. Isobars are nuclides that share the same mass number, A. Many times there are no isobars; that is, there is only one stable nuclide for a given mass. The enlarged section begins this way. For mass 27 there is only ^{27}Al. For mass 28 there is only ^{28}Si. For mass 30 there is only ^{30}Si. There are two isobars for masses 36, 40, 46 and 48. Notice that we did not count three isobars for mass 40 because ^{40}K is not truly stable. In general, there is only one or two nuclides that is/are stable for each mass number. There are just a couple instances where there are three. Also note that when there are two, there is a gap between them. There is ^{36}Ar and ^{36}S but ^{36}Cl is not stable. Likewise ^{40}K, ^{46}Sc, and ^{48}Sc are each unstable nuclides sandwiched between two stable isobars.

To explain this behavior we begin a more detailed study of nuclear stability versus beta decay. We are going to make the following assumptions:

1) There are quantum levels in the nucleus for both protons and neutrons.
2) The quantum levels for protons are a little higher in energy because of Coulomb repulsion.
3) Each quantum level can hold a specific number of nucleons.
4) Protons and neutrons, like electrons, have spin. It is energetically favorable for nucleons to fill orbitals in pairs of opposite spins. In general, protons pair with protons and neutrons pair with neutrons.
5) If it is energetically favorable for a neutron to switch to a proton, it can do so via beta-minus decay. Likewise, if it is energetically favorable for a proton to switch to a neutron, it can do so by electron-capture or beta-plus decay.
6) $E = mc^2$, and this means "energetically favorable" can be determined from nuclide masses.

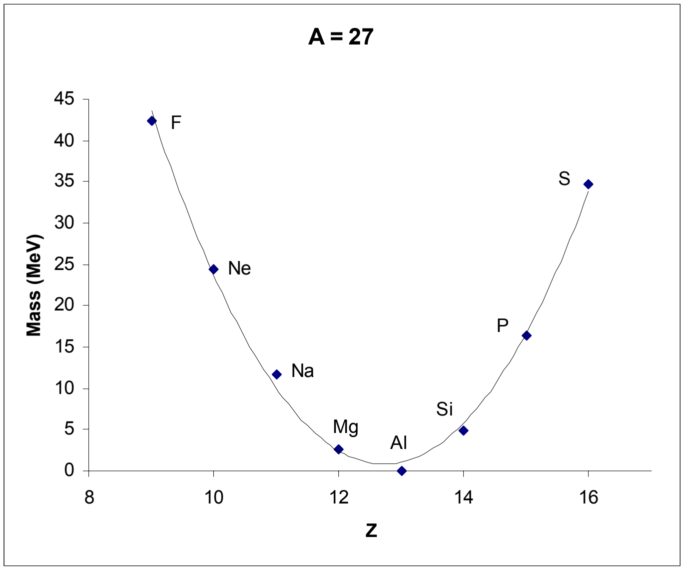
Let's again look at an enlarged portion of the Chart of the Nuclides in the region of atomic mass of about 30. This time we add a few known unstable nuclides. The nuclides in the bold boxes are stable, and those in non-bold boxes decay by beta decay. The only non-stable nuclides added are those having mass numbers of 27 and those having mass numbers of 36.



On the chart, isobars run diagonally high to low as we go left to right. The A = 27 isobar runs from proton-rich ^{27}S on the upper left to neutron-rich ^{27}F on the lower right. Of the eight known nuclides having A = 27, only aluminum is stable. You can also see that there is only one stable nuclide for mass 23, 24, 25, etc. all the way to mass 36. For A = 36 there are nine known nuclides, and two of them (^{36}Ar and ^{36}S) are stable. The others undergo beta decay. What's going on?

The nuclide masses have been measured for many nuclides, and these masses can also be calculated using a formula known as the *semi-empirical mass formula*. The mass of ^{27}Al is 26.9815384 amu. Let's subtract that mass from the masses of the other nuclides having A = 27, and let's plot the mass differences on a graph of mass versus Z. Let's also use $E = mc^2$ and use units of MeV for the mass differences.

All the other A = 27 nuclides are more massive than aluminum, and they become more massive the further they are from aluminum's Z of 13. This graph of energy versus atomic number for A = 27 appears on the next page.



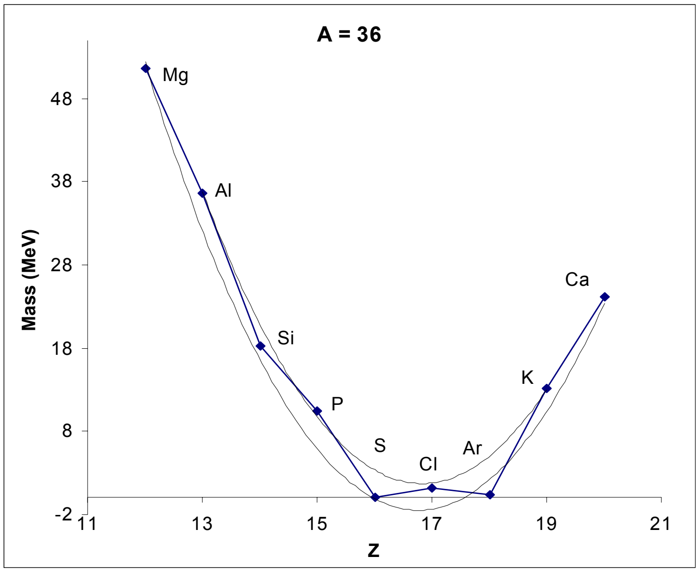
The plot is almost a perfect parabola with aluminum at the bottom. Looking on the left side of aluminum we see that if ^{27}F would beta-minus decay to ^{27}Ne, the mass (energy) would go down by almost 20 MeV. The actual number is 18.0 MeV and this is called Q-beta-minus. A reasonable way to interpret this is that ^{27}Ne is more stable than ^{27}F to the tune of about 18.0 MeV because 18.0 MeV of energy will be "released" when this beta decay occurs. The energy mostly appears as kinetic energy of the beta and antineutrino. There is also a bit of kinetic energy for the nucleus as it recoils. Also, the beta-decay might not leave the ^{27}Ne in the ground state. If that occurs, some of the 18.0 MeV of energy stays in the nucleus and later comes out in gamma radiation.

On the other side, ^{27}S can beta-plus decay to ^{27}P and release energy of about 20 MeV. The actual energy is 18.3 MeV and this is called Q-electron-capture. It is not called Q-beta-plus because 1.02 MeV of the 18.3 MeV available has to be used to create the electron and the positron in order to allow the beta decay to proceed. The mass of an electron or positron is 511 keV = 0.511 MeV and we have to create one of each, so we need a minimum of 1022 keV or 1.022 MeV of energy for beta-plus to proceed. That means the energy shared as kinetic energy of beta-plus, neutrino, and nuclear recoil can at most be 18.3 Mev minus 1.022 MeV or about 17.3 MeV. We could say Q-EC is 18.3 and Q-β^+ is 17.3, but we generally use the Q-EC notation. Once you know what is going on, there is no mistake that Q-EC means the total energy difference between the two nuclides involved in the beta decay.

Notice that the data do not fit a perfect parabola. Starting with ^{27}F and moving toward ^{27}S we alternate back and forth between odd-even and even-odd for Z-N. ^{27}F is odd-even, ^{27}Ne is even-odd, ^{27}Na is odd-even, ^{27}Mg is even-odd, and so forth. This makes the actual nuclide masses weave around the parabola a bit.

You can draw a similar parabola for every isobar on the Chart of the Nuclides. The nuclide on the bottom is stable with respect to beta decay. How could ^{27}Al beta decay? If it were to undergo beta-minus it would become ^{27}Si and that would require energy rather than release energy. If it were to undergo beta-plus or electron capture, it would become ^{27}Mg and that also would required energy.

If this is the general trend for beta decay, how can there be two stable nuclides for mass 36? Any even-mass nuclide has the interesting property that Z-N must either be even-even or odd-odd. Therefore, when you travel up or down the isobar you are alternating, E-E, O-O, E-E, O-O, etc. Compare this to odd-mass isobars where you alternate E-O, O-E, E-O, O-E, etc. E-E is really good, all protons and all neutrons are paired. O-O is really bad, one proton and one neutron are not paired. Above we said that the E-O, O-E alternation lead to some weaving around the parabola. The weaving is so major with an even-mass isobar that we essentially have two parabolas. Let's look at the A = 36 isobar.



As seen in the graph, the odd-odd isobars fall on a parabola higher in mass than the even-even isobars. If it weren't for the odd-odd problem (i.e. if it weren't for lack of pairing) we would expect ^{36}Cl as the stable A = 36 nuclide. But the even-even nature of ^{36}S and ^{36}Ar have lowered them below the ^{36}Cl. In this case ^{36}S has the lowest mass, so it clearly has nothing to decay to, and it is stable. ^{36}Ar is about 0.434 Mev higher than ^{36}S, but it can't beta decay directly to ^{36}S because it would have to go first to ^{36}Cl which it can't do because ^{36}Cl is higher in energy.

So we have the interesting situation that there are two stable A = 36 nuclides. Another interesting outcome is that ^{36}Cl appears to have a choice of doing a beta-minus to ^{36}Ar or a beta-plus or electron-capture to ^{36}S. The beta-plus almost cannot occur because the energy difference between ^{36}Cl and ^{36}S is only 1.14 MeV and that is just barely enough energy to create the positron-electron pair needed for beta-plus. Therefore, if the ^{36}Cl is going to decay to ^{36}S it is generally going to be EC. On the other hand. EC probabilities can be low because an atomic electron must have some probability of overlap with the nucleus in order for the electron to be captured. The bottom line is that it is easier for ^{36}Cl to beta-minus to ^{36}Ar even though the energy difference there of 0.712 MeV is less than the 1.14 MeV if it were to go to ^{36}S. In the end, the ratios for ^{36}Cl decay are: 98.1% beta-minus to ^{36}Ar, 1.9% EC to ^{36}S, and 0.001% beta-plus to ^{36}S.

It is also possible for their to be three stable nuclides for an isobar. This happens for A = 124 where we have the even-even nuclides of ^{124}Xe, ^{124}Te, and ^{124}Sn each surrounded by odd-odd nuclides with more mass (just like S and Ar are surrounded in the graph above. In the case of A = 124 the odd-odd nuclides between ^{124}Xe, ^{124}Te, and ^{124}Sn are ^{124}I and ^{124}Sb, so ^{124}I and ^{124}Sb (like ^{36}Cl) can both beta-minus and beta-plus decay. In the 2003 Chart of the Nuclides there are 33 nuclides that can decay by both beta-plus and beta-minus.